LIBERTY PAPER SET

STD. 10 : Mathematics (Standard) [N-012(E)]

Full Solution

Time: 3 Hours

ASSIGNTMENT PAPER 3

Section-A

1. (A) 4 **2.** (B) $\frac{800}{x+20}$ **3.** (C) 5 **4.** (B) 70° **5.** (B) $2\pi r^2$ **6.** (D) 3 **7.** 18 **8.** - 8 **9.** 2 **10.** 8.4 **11.** (*a*, *a*) **12.** $\pi r^2 h$ **13.** False **14.** True **15.** False **16.** True **17.** 20 - 30 **18.** 0.9 **19.** 4 **20.** $ax^2 + bx + c = 0$, $a \neq 0$ **21.** (b) $\frac{c}{a}$ **22.** (c) $-\frac{d}{a}$ **23.** (b) 1 **24.** (c) - 1

Section-B

25. Let, $5 + 2\sqrt{7}$ be rational.

Therefore, $5 + 2\sqrt{7} = \frac{a}{b}$, where *a* and *b* are integers having no common factors other then 1. Noe, $5 + 2\sqrt{7} = \frac{a}{b}$, $\Rightarrow 2\sqrt{7} = \frac{a}{b} - 5$ $\Rightarrow 2\sqrt{7} = \frac{a - 5b}{b}$ $\Rightarrow \sqrt{7} = \frac{a - 5b}{2b}$

Since, a and b are integers, therefore, $\sqrt{7}$ is rational.

This contradicts the fact that $\sqrt{7}$ is irrational.

Therefore, our assumption was wrong

Hence, $5 + 2\sqrt{7}$ is irrational.

26. Here,
$$\frac{4}{3} = x + 2y = 8$$

and $2x + 3y = 12$
 $\frac{4}{3} = x + 2y - 8 = 0$
and $2x + 3y - 12 = 0$
 $a_1 = \frac{4}{3}, b_1 = 2, c_1 = -8$
 $a_2 = 2, b_2 = 3, c_2 = -12$
Now, $\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3}, \frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$
Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given pair of liners equations is consistent and dependent.

27. Suppose, the size of the base = x cm Hence, the measurement of altitude = (x - 7) cm

According to Pythagoras theorem,

 $(Base)^2 + (Altitude)^2 = (Hypotenuse)^2$

 $\therefore (x)^{2} + (x - 7)^{2} = (13)^{2}$ $\therefore x^{2} + x^{2} - 14x + 49 = 169$ $\therefore 2x^{2} - 14x + 49 - 169 = 0$ $\therefore 2x^{2} - 14x - 120 = 0$ $\therefore x^{2} - 7x - 60 = 0$ $\therefore x^{2} - 12x + 5x - 60 = 0$ $\therefore x(x - 12) + 5(x - 12) = 0$ $\therefore (x + 5)(x - 12) = 0$ $\therefore x + 5 = 0 \quad \text{OR} \quad x - 12 = 0$ $\therefore x = -5 \quad \text{OR} \quad x = 12$

But the size of the base should not be negative.

The base of the given triangle = 12 cm

The altitude of this triangle will be = 12 - 7 = 5 cm.

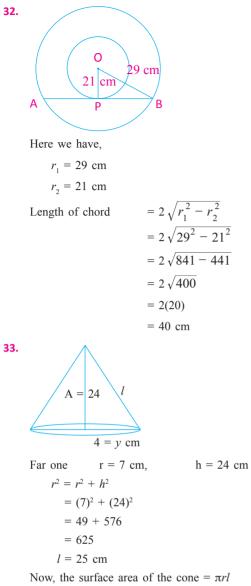
28.
$$3x^2 - 2x + \frac{1}{3} = 0$$

 $\therefore 9x^2 - 6x + 1 = 0$
 $\therefore a = 9, b = -6, c = 1$
 $\therefore b^2 - 4ac = (-6)^2 - 4(9)(1) = 36 - 36 = 0$
 $\therefore b^2 - 4ac = 0$, the given equdratic equation has two equal real roots.
 $\therefore x = -\frac{b}{2a} = -\frac{(-6)}{2(9)} = \frac{6}{18} = \frac{1}{3}$
The roots are : $\frac{1}{3}, \frac{1}{3}$
29. Here, $a = 7, d = 13 - 7 = 6$
 $a_n = a + (n - 1) d$
 $a_{20} = a + (20 - 1) d$
 $= a + 19d$
 $= 7 + 19 (6)$
 $= 7 + 114$
 $a_{20} = 121$
26. $A = 2x - 2450 \pm x - 50 \pm 2600 = 2$

$$\therefore 2 (1)^2 + x - \left(\frac{\sqrt{3}}{2}\right)^2 = 2$$

$$\therefore 2 (1)^2 + x - \left(\frac{1}{2}\right)$$
$$\therefore 2 (1) + x - \frac{3}{4} = 2$$
$$\therefore 2 + x - \frac{3}{4} = 2$$
$$\therefore x = 2 - 2 + \frac{3}{4}$$
$$\therefore x = \frac{3}{4}$$

31. $2 \sin \theta + \cos \theta = 2$ \therefore 2 sin θ = 2 - cos θ $\therefore (2 \sin \theta)^2 = (2 - \cos \theta)^2$ $\therefore 4 \sin^2 \theta = 4 - 4 \cos \theta + \cos^2 \theta$ $\therefore 4 (1 - \cos^2 \theta) = 4 - 4 \cos \theta + \cos^2 \theta$ $\therefore \quad 4 - 4 \cos^2 \theta = 4 - 4 \cos \theta + \cos^2 \theta$ $\therefore 4 - 4 \cos^2 \theta - 4 + 4 \cos \theta - \cos^2 \theta = 0$ $\therefore -5 \cos^2 \theta + 4 \cos \theta = 0$ \therefore 5 cos² θ - 4 cos θ = 0 $\therefore \cos \theta (5 \cos \theta - 4) = 0$ $\therefore \cos \theta = 0 \text{ OR } 5 \cos \theta - 4 = 0$ $\therefore \cos\theta = 0 \text{ OR } \cos\theta = \frac{4}{5}$ $(\cos\theta = 0)$ Not possible $\therefore \cos \theta = \frac{4}{5}$...(1) $\therefore \quad \cos^2 \theta = \frac{16}{25}$ $\therefore \quad 1 - \sin^2 \theta = \frac{16}{25}$ $\therefore \quad \sin^2 \theta = 1 - \frac{16}{25}$ $\therefore \quad \sin^2 \theta = \frac{25 - 16}{25}$ $\therefore \quad \sin^2 \theta = \frac{9}{25}$ \therefore sin $\theta = \frac{3}{5}$...(2) $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $= \frac{\frac{3}{5}}{\frac{4}{5}}$ $= \frac{3}{4}$ or Here $2 \sin \theta + \cos \theta = 2$ $\therefore \quad \frac{2\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} = \frac{2}{\cos\theta}$ (: divide by $\cos \theta \neq 0$) $\therefore 2tan + 1 = 2sec \theta$ $\therefore (2tan \ \theta + 1)^2 = 4sec^2 \ \theta$ (∴ Square both side) $\therefore 4tan^2\theta + 4tan \ \theta + 1 = 4 \ (1 + tan^2 \ \theta)$ $\therefore 4tan^2\theta + 4tan \ \theta + 1 = 4 + 4tan^2 \ \theta$ $\therefore 4tan\theta = 4 + 4tan^2 \theta - 4tan^2 \theta - 1$ $\therefore 4tan\theta = 3$ $\therefore tan\theta = \frac{3}{4}$



- $= \frac{22}{7} \times 7 \times 25$ $= 550 \text{ cm}^2$
- **34.** Here, maximum frequency is 7 which belongs to class 40 55. Hence, 40 55 is the modal class. $\therefore l =$ lower limit of the modal class = 40
 - h = class size = 15
 - f_1 = frequency of the modal class = 7
 - f_0 = frequency of the class preceding the modal class = 3
 - f_2 = frequency of the class succeeding the modal class = 6

Mode
$$Z = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

= $40 + \left(\frac{7 - 3}{2(7) - 3 - 6}\right) \times 15$
= $40 + \frac{4 \times 15}{14 - 9}$
= $40 + \frac{4 \times 15}{5}^3$
= $40 + 12$
 $\therefore Z = 52$

class	f_{i}	<i>x</i> _i	u _i	$f_i \mu_i$
11 – 13	7	12	-2	-14
13 – 15	6	14	-1	-6
15 – 17	f	16 = <i>a</i>	0	0
17 – 19	13	18	1	13
19 – 21	20	20	2	40
21 – 23	5	22	3	15
23 - 25	4	24	4	16
Total	$\Sigma f_i = f + 55$		-	$\Sigma f_i u_i = -20 + 84 = 64$

$$a = 16, h = 2$$

- Mean $x = a + \frac{\sum f_i u_i}{\sum f_i} \times h$ $\therefore 18 = 16 + \frac{64}{f + 55} \times 2$ $\therefore 18 - 16 = \frac{64 \times 2}{55}$ $\therefore 2 = \frac{64 \times 2}{f + 55}$ $\therefore f + 55 = \frac{64 \times 2}{2}$ $\therefore f + 55 = 64$ $\therefore f = 64 - 55$ $\therefore f = 9$
 - \therefore The missing frequency f = 9
- 36. Out of the two friends, one girl, say, Salma's birthday can be any day of the year. Now, Mona's birthday can also be any day of 365 days in the year.

We assume that these 365 outcomes are equally likely.

- (i) If Mona's birthday is different from Salma's, the number of favourable outcomes for her birthday is 365 1 = 364So, P (Mona's birthday is different from Salma's birthday) = $\frac{364}{365}$

(ii) P (Salma and Mona have the same birthday)

= 1 - P (both have different birthdays)

$$= 1 - \frac{364}{365} = \frac{1}{365}$$

- 37. Two dice are thrown same time, then the result are following :
 - (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \therefore Total number of outcomes = 36

(i) Suppose A be the event "the number of two dice are same"

There are 6 results (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6) for the event

 \therefore The number of outcomes favourable to

:. P (A) = $\frac{6}{36} = \frac{1}{6}$

Section-C

38. Suppose
$$\alpha = 2 + \sqrt{3}$$
, $\beta = 2 - \sqrt{3}$
 $\therefore \quad \alpha + \beta = 2 + \sqrt{3} + 2 \quad \sqrt{3} = 4$
 $\alpha \cdot \beta = (2 + \sqrt{3}) \quad (2 - \sqrt{3}) = 4 - 3 = 1$

... The requird quadratic polynomial

$$= k [x^{2} - (\alpha + \beta) x + \alpha \beta], k \neq 0, k \in \mathbb{R}$$
$$= k [x^{2} - 4x + 1]$$

39. Here $6x^2 - 13x + 6 = 0$

- $\therefore 6x^2 9x 4x + 6 = 0$
- \therefore 3x (2x 3) 2 (2x 3) = 0
- $\therefore (2x-3)(3x-2) = 0$
- $\therefore 2x 3 = 0 \quad \text{OR} \quad 3x 2 = 0$
- $\therefore 2x = 3$ OR 3x = 2
- $\therefore x = \frac{3}{2}$ OR $x = \frac{2}{3}$
- 2 2

Suppose $\alpha = \frac{3}{2}$, $\beta = \frac{2}{3}$

$$a = 6, b = -13, c = 6$$

Sum of zeros $(\alpha + \beta) = \frac{3}{2} + \frac{2}{3} = \frac{9+4}{6} = \frac{(-13)}{6} = \frac{-b}{a} = \frac{-\text{ coefficient of } x}{\text{ coefficient of } x^2}$

...(2)

Product of zeros $(\alpha + \beta) = \left(\frac{3}{2}\right) \left(\frac{2}{3}\right) = \frac{6}{6} = \frac{c}{a} = \frac{\text{coefficient term}}{\text{coefficient of } x^2}$

40. Here for first AP, 65, 67, 69,...

$$\therefore A = 65, D = 67 - 65 = 2$$

$$\therefore An = A + (n - 1)D$$

$$\therefore An = 65 + (n - 1)(2)$$

$$\therefore An = 65 + 2n - 2$$

$$\therefore An = 63 + 2n$$
...(1)

for second AP, 10, 17, 24,...

a = 10, d = 17 - 10 = 7∴ an = a + (n - 1)d∴ an = 10 + (n - 1)7∴ an = 10 + 7n - 7

$$\therefore$$
 an = 3 + 7n

compare (1) & (2) An = an63 + 2n = 3 + 7n2n - 7n = 3 - 63-5n = -605n = 60 $n = \frac{60}{5}$ n = 12 So, n = 12 term will be equal in both APs. **41.** Here a = -2, d = -5 - (-2) = -5 + 2 = -3, $a_n = -227$ we have, $a_n = a + (n - 1) d$ \therefore -227 = -2 + (n - 1)(-3) \therefore -227 + 2 = (n - 1)(-3) $\therefore -225 = (n-1)(-3)$ $\therefore \quad \frac{-225}{-3} = n - 1$

- :. n 1 = 75 $\therefore n = 75 + 1$ n = 76Here, $a_n = 1 = -227$ $\mathrm{S}n = \frac{n}{2}(a+1)$ \therefore S76 = $\frac{76}{2} [-2+(-227)]$ \therefore S76 = 38(- 229)
 - \therefore S76 = 8702
- 42. Suppose, A (1, 2), B (4, y), C (x, 6) and D (3, 5) are the vertices of parallelogram ABCD.

Co-ordinates from the midpoint of the diagonal AC

= Co-ordinates from the midpoint the diagonal BD.

$$\therefore \left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{4+3}{2}, \frac{y+5}{2}\right)$$

$$\therefore \frac{1+x}{2} = \frac{4+3}{2} , \frac{2+6}{2} = \frac{y+5}{2}$$

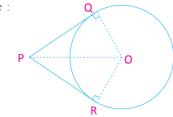
$$\therefore 1+x=7 , 8=y+5$$

$$\therefore x=7-1 , y=8-5$$

$$\therefore x=6 , y=3$$

43. Given : A circle with centre O, a point P lying outside the circle with two tangents PQ, QR on the circle from P.

To prove : PQ = PRFigure :



Proof : Join OP, OQ and OR. Then \angle OQP and \angle ORP are right angles because these are angles between the radii and tangents and according to theorem 10.1 they are right angles.

Now, in right triangles OQP and ORP,

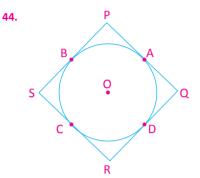
OQ = OR (Radii of the same circle)

OP = OP (Common)

 $\angle OQP = \angle ORP$ (Right angle)

Therefore, \triangle OQP $\cong \triangle$ ORP (RHS)

This gives, PQ = PR (CPCT)



Here sides of given quadrilateral touch circle at points A, B, C, D with. PQ, PS, SR & RQ respectively.

÷.	PA = PB	(1)
	QA = QD	(2)
	RC = RD	(3)
	SC = SB	(4)

Add (1), (2), (3), (4) results,

PA + QA + RC + SC = PB + QD + RD + SB

- $\therefore (PA + QA) + (RC + SC) = (QD + RD) + (PB + SB)$
- \therefore PQ + RS = QR + PS

Hence proved.

45.
$$r = 14 \text{ cm}$$

Total angle = 360°

 $\therefore \text{ Angle for 15 minute duration} = \frac{360 \times 3}{12}$ $= 90^{\circ}$ $\therefore \text{ Area during 15 mins} = \frac{\pi r^2 \theta}{360}$ $= \frac{22 \times 14 \times 14 \times 90}{7 \times 360}$ $= \frac{2 \times 11 \times 7 \times 2 \times 14}{7 \times 4}$ $= 22 \times 7$ $= 154 \text{ cm}^2$

Remaining area to complete 1 circle

$$= \pi r^{2} - \text{Area in 15 mins}$$

$$= \left(\frac{22}{7} \times 14 \times 14\right) - 154$$

$$= (22 \times 2 \times 14) - 154$$

$$= 616 - 154$$

$$= 462 \text{ cm}^{2}$$

- **46.** Total number of balls = 7 green + 5 yellow + 8 brown = 20
 - (i) Suppose A be the event "Selected ball is green"

Numbers of green balls = 7

 \therefore The number of outcomes favourable to A = 7

$$\therefore$$
 P (A) = $\frac{7}{20}$

- (ii) Suppose B be the event "Selected ball is brown" Numbers of brown balls = 8
- \therefore The number of outcomes favourable to B = 8

:. P (B) =
$$\frac{8}{20} = \frac{2}{5}$$

(iii) Suppose C be the event "Selected ball is not a yellow"

Numbers of yellow balls = 5

- \therefore Number of without yellow balls = 20 5 = 15
- \therefore The number of outcomes favourable to C = 15

: P (C) =
$$\frac{15}{20} = \frac{3}{4}$$

Section-D

47. Suppose, the larger number is x and the smaller number is y. The difference of two positive integers is 5. $\therefore x - y = 5$ (1)

and the difference of their inverse is $\frac{1}{10}$ $\therefore \frac{1}{y} - \frac{1}{x} = \frac{1}{10}$(2)(1) As per equation y = x - 5....(3) put value of eqn. (3) in equ. (2) $\therefore \frac{1}{x-5} - \frac{1}{x} = \frac{1}{10}$ $\therefore 10 \ x - 10 \ (x - 5) = x \ (x - 5)$ $\therefore 10 \ x - 10 \ x + 50 = x^2 - 5x$ $\therefore 50 = x^2 - 5x$ $\therefore x^2 - 5x - 50 = 0$ $\therefore x^2 - 10 x + 5 x - 50 = 0$ $\therefore x (x - 10) + 5 (x - 10) = 0$ $\therefore (x - 10) (x + 5) = 0$ $\therefore x - 10 = 0$ OR x + 5 = 0OR x = -5 $\therefore x = 10$

but x is a positive integes then negative is not possible.

 $\therefore x \neq -5$ $\therefore x = 10$ Put x = 10 in equation (3) $\therefore y = 10 - 5$ $\therefore y = 5$ Hence, required two numbers are 10 and 5. **48.** (a) Let, x + y = 4 ...(1) 2x = 8 + 3y 2x - 3y = 8 ...(2) From equation (1), x = 4 - y ...(3)

Put value of quation (3) in eq. (2)

2(4-y)-3y=8

 $\therefore 8 - 2y - 3y = 8$

$$\therefore -5y = 8 - 8$$

 $\therefore -5y = 0$

 $\therefore y = 0$

Put y = 0 in equation (3)

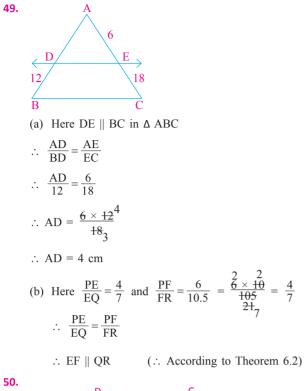
$$x = 4 - 0$$

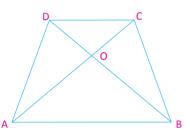
x = 4

$$x = 4, y = 0$$

(b) The \square ABCD is cyclic quadriateral then the opoosite angle of them is complementary.

$$\angle A + \angle C = 180^{\circ} 2x - y + x - 2y = 180^{\circ} 3x - 3y = 180^{\circ} x - y = 60^{\circ} ...(1) and $\angle B + \angle D = 180^{\circ} 2y + x = 180^{\circ} ...(2) Do equ : (2) - (1) x + 2y = 180^{\circ} x - y = 60^{\circ} -+ - 3y = 120^{\circ} y = 40^{\circ} From (1) x - 40^{\circ} = 60^{\circ} x = 60^{\circ} + 40^{\circ} x = 100^{\circ} Hence x = 100^{\circ} and y = 40^{\circ}$$$





Here, ABCD is a trapezium where AB || DC. $\therefore \angle CAB = \angle ACD \text{ and } \angle DBA = \angle BDC$ Now, in $\triangle OAB$ and $\triangle OCD$, $\angle OAB = \angle OCD \text{ and } \angle OBA = \angle ODC$ $\therefore \triangle OAB \sim \triangle OCD$ $\therefore \frac{AO}{CO} = \frac{BO}{DO}$ $\therefore \frac{AO}{BO} = \frac{CO}{DO}$

51. A 30° 45° F 30° D B 45° C

...(1)

Here, AB = 100 m \angle FAD = \angle ADE = 30° \angle FAC = \angle ACB = 45° In $\triangle ABC$, $\therefore \quad tan \ 45^\circ = \frac{AB}{BC}$ $\therefore \quad 1 = \frac{100}{BC}$ ÷ BC = 100 m.In $\triangle AED$, \therefore tan 30° = $\frac{AE}{ED}$ $\therefore \frac{1}{\sqrt{3}} = \frac{AE}{BC} (\because ED = BC)$ $\therefore \frac{1}{\sqrt{3}} = \frac{AE}{100}$ \therefore AE = $\frac{100}{\sqrt{3}}$ \therefore AE = 0.58 × 100 ∴ AE = 58 m Here, as per figure, A - E - BAE + EB = AB \therefore 58 + CD = 100 (\therefore EB = CD) \therefore CD = 100 - 58 \therefore CD = 42 m

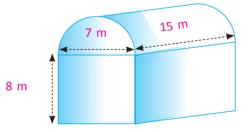
Height of tower is 42 m

Distance between cliff & tower is 100 m.

52. Cuboid Half Cylinder l = 7 m $h_1 = \text{Breadth of cuboid} = 15 \text{ m}$ b = 15 m d = Length of cuboid = 7 mh = 8 m $\therefore r_1 = \frac{7}{2} \text{ m}$

The required volume

= Volume of cuboid + $\frac{1}{2}$ Volume of cylinder = $lbh + \frac{1}{2} \pi r_1^2 h_1$ = $(7 \times 15 \times 8) + \left(\frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 15\right)$ = $840 + \frac{1155}{4}$ = 840 + 288.75= 1128.75 m^3



Now, the total space occupied by the machinery = 300 m³ and the total space occupied by the workers = $20 \times 0.08 = 1.6$ m³ Therefore, the volume of the air, when there are machinery and workers = 1128.75 - 300 - 1.6 = 827.15 m³

53.

$$28 \text{ cm}$$

$$26 \text{ cm}$$

$$d = 28 \text{ cm}$$

$$\therefore r = \frac{28}{2} = 14 \text{ cm}$$

$$h = \text{total height} - r$$

$$\therefore h = 26 - 14$$

$$\therefore h = 12 \text{ cm}$$
Inner surface area of vessel
$$= \text{curved surface of cylinder + curved surface of hemisphere}$$

$$= 2\pi r h + 2\pi r^2$$

$$= 2\pi r (h + r)$$

$$= 2\pi r (h + r)$$

= 2 × $\frac{22}{7}$ × 14 × (12 + 14)
= 44 × 2(26)
= 2288 cm²

Inner surface area of vessel is 2288 cm².

54.

Class	Frequency	Cumulative frequency
20 - 30	15	15
30 - 40	16	15 + 16 = 31
40 - 50	38	31 + 38 = 69
50 - 60	15	69 + 15 = 84
60 - 70	9	84 + 9 = 93
70 - 80	7	93 + 7 = 100
	<i>n</i> = 100	

Here,
$$n = 100 \frac{n}{2} = \frac{100}{2} = 50$$

50 lies with cf value 69

:. class =
$$40 - 50$$
, $l = 40$, $cf = 31$,

$$h = 10, f = 38$$

Median (M)=
$$l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

= $40 + \left[\frac{50 - 31}{38}\right] \times 10$
= $40 + \left[\frac{19 \times 10}{19 \times 2}\right] \times 10$
= $40 + 5$
M = 45

Median of given data is 45 years.